Math 1B Discussion Problems 12 Feb

1. (a) Estimate the integral $\int_0^1 \sin(x^2)dx$ using (i) the Midpoint Rule, (ii) the Trapezoidal Rule, (iii) Simpson's Rule, each with $n = 6$.

   (b) How large do we have to pick n to ensure an error of < 0.001, under each of the approximation rules we used?

2. Evaluate the following integrals.

   (a) $\int_0^\infty \frac{1}{x^2+1}dx$

   (b) $\int_0^\infty \frac{1}{x^2-1}dx$

   (c) $\int_0^\infty e^{-x}\sin x dx$

3. * For each of the following statements, decide whether it is true or false; if it is true, give a reason, if it is false, give a counterexample.

   (a) If $\lim_{x\to\infty} f(x) = 0$, then $\int_1^\infty f(x)dx$ converges.

   (b) If $\int_1^\infty f(x)dx$ converges, then $\lim_{x\to\infty} f(x) = 0$.

   (c) If $\int_1^\infty f(x)dx$ diverges, then $\lim_{x\to\infty} f(x) = \infty$.